RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2018-21]
B.A./B.Sc. SECOND SEMESTER (January – June) 2019
Mid-Semester Examination, March 2019

Date : 25—03—2019

MATHEMATICS (Honours)

Time : 11 am – 1 pm

Paper : II

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

[25 marks]

1. Answer **any one** question: [1×5]
   a) State Descartes' rule of signs.
      Apply this rule to find the nature of the roots of the equation: $x^4+2x^2+3x-1=0$.
   b) Solve the equation : $3x^6+x^5-27x^4+27x^2-x-3=0$.

2. Answer **any two** questions: [2×4]
   a) z is a variable complex number such that $\left|z-\frac{10}{z}\right|=3$. Find the greatest and the least value of $|z|$.
   b) Prove that $2^8\sin^9\theta=\sin 9\theta-9\sin 7\theta+36\sin 5\theta-84\sin 3\theta+126\sin\theta$.
   c) Show that the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(\log 2)+i\cos(\log 2)$.

3. Answer **any three** questions : [3×4]
   a) State and prove Leibnitz test for alternating series.
   b) Show that in absolutely convergent series rearrangement of the terms does not affect the convergence.
   c) "A subset K of $\mathbb{R}$ is said to be compact if every open cover of K has a finite subcover". Using this definition, show that, if F is a closed subset of a compact set K in $\mathbb{R}$, then F is compact.
   d) Let $f:[a,b]\to\mathbb{R}$ be a function with the property that for every $x\in[a,b]$, the function is bounded on a neighbourhood $N(x,\delta_x)$ of x. Prove that f is bounded on [a,b].
   e) Let $f:[a,b]\to\mathbb{R}$ be continuous on [a,b]. Show that f is bounded on [a,b].

Group – B

[25 marks]

4. Express $\begin{bmatrix}1 & \alpha & 1\\ \beta & 1 & 1\\ 1 & 1 & \gamma\end{bmatrix}$ as the sum of symmetric and a skew symmetric matrices. [2]

5. Answer **any two** questions : [2×4]
   a) Solve $\begin{vmatrix}11-x & -6 & 2\\ -6 & 10-x & -4\\ 2 & -4 & 6-x\end{vmatrix}=0$

b) For a fixed positive integer n, if

$$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix} \text{ then show that } \frac{D}{(n!)^3} - 4 \text{ is divisible by n.}$$

c) Solve by Cramer's rule:

$$x + y + z = 1$$
$$ax + by + cz = 1$$
$$abx + bcy + caz = 1 \quad \text{where } a, b, c \text{ are unequal.}$$

Answer <u>any one</u> question from question nos.6 & 7 **:** [1×15]

6. a) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b,\ x \geq 0$ corresponds to a B.F.S. [5]

b) Solve the following minimization transportation problem, using matrix minima method to get initial B.F.S. : [6]

	A	B	C	a_i
I	6	8	4	14
II	4	9	3	12
III	1	2	6	5
b_i	6	10	15	

c) Find all the basic solutions of the followings equations identifying in each case the basic vectors and the basic variables: [4]

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 4.$$

7. a) If the L.P.P. Maximize $z = cx$, subject to $Ax = b,\ x \geq 0$; admits of an optimal solution, then prove that the optimal solution, will coincide with at least one B.F.S. [7]

b) Five operators have to be assigned to five machines. The assignment costs are given below. Operator A cannot operate machine III and operator C cannot operate machine IV. Find the optimal assignment schedule. [5]

	I	II	III	IV	V
A	5	5	-	2	6
B	7	4	2	3	4
C	9	3	5	-	3
D	7	2	6	7	2
E	6	5	7	9	1

c) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ of the system of equations

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8$$

to a B.F.S. [3]

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